MAHARASHTRA STATE BOARD OF TECHNICAL EDUCATION

(Autonomous)

(ISO/IEC - 27001 - 2013 Certified)

WINTER – 2018 EXAMINATION

Subject Name: Applied Mathematics <u>Model Answer</u>

Subject Code:

22206

Important Instructions to Examiners:

- 1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
- 2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
- 3) The language errors such as grammatical, spelling errors should not be given more importance (Not applicable for subject English and Communication Skills).
- 4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
- 5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answer and model answer.
- 6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
- 7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.		Attempt any FIVE of the following:	10
	a)	Test whether the function is even or odd if $f(x) = x^3 + 4x + \sin x$.	02
	Ans	$f(x) = x^3 + 4x + \sin x$	1/2
		$\therefore f(-x) = (-x)^3 + 4(-x) + \sin(-x)$ $= -x^3 - 4x - \sin x$	1/2
		$= -\left(x^3 + 4x + \sin x\right)$	1/2
		=-f(x)	1/2
		∴ function is odd.	
	b)	If $f(x) = x^2 + 5x + 1$ then find $f(0) + f(1)$	02
	Ans	$f(x) = x^2 + 5x + 1$	
		$\therefore f(0) = (0)^2 + 5(0) + 1 = 1$	1/2
		$\therefore f(1) = (1)^2 + 5(1) + 1 = 7$	1/2
		$\therefore f(0) + f(1) = 1 + 7 = 8$	1
	c)	Find $\frac{dy}{dx}$ If $y = x^n + a^x + e^x + \sin x$	02
	Ans	$y = x^n + a^x + e^x + \sin x$	
		$\therefore \frac{dy}{dx} = nx^{n-1} + a^x \log a + e^x + \cos x$	1/2+1/2
		dx	+1/2+1/2
	d)	Evaluate $\int xe^x dx$	02



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No. Q. N.	Q.	Sub		Marking
Ans $ \begin{vmatrix} xe^{x}dx = x \\ = xe^{x} - \int (e^{x}.1)dx \\ = xe^{x} - e^{x} + c \end{vmatrix} $ e) $ \begin{aligned} Evaluate \int_{0}^{x} \tan^{2}x dx \\ = \int_{0}^{x} (\sec^{2}x - 1) dx \\ = \tan x - x + c \end{aligned} $ f) $ \begin{aligned} Area A = \int_{0}^{x} y dx \\ & \therefore A = \begin{bmatrix} 3^{2} \\ 2 \end{bmatrix}^{3} \end{aligned} $ or $ A = \begin{bmatrix} x^{2} \\ 3 \end{bmatrix}^{3} $ or $ A = \begin{bmatrix} x^{2} \\ 3 \end{bmatrix}^{3} $ $ A = \begin{bmatrix} x - 1^{2} \\ 2 \end{bmatrix} $ or $ A = \begin{bmatrix} 3^{2} - 1^{2} \\ 2 \end{bmatrix} $ or $ A = \begin{bmatrix} 3^{2} - 1^{2} \\ 2 \end{bmatrix} $ or $ A = \begin{bmatrix} 3^{2} - 1^{2} \\ 2 \end{bmatrix} $ or $ A = \begin{bmatrix} 3^{2} - 1^{2} \\ 2 \end{bmatrix} $ or $ A = \begin{bmatrix} 3^{2} - 1^{2} \\ 2 \end{bmatrix} $ or $ A = \begin{bmatrix} 3^{2} - 1^{2} \\ 2 \end{bmatrix} $ or $ A = \begin{bmatrix} 3^{2} - 1^{2} \\ 2 \end{bmatrix} $ or $ A = \begin{bmatrix} 3^{2} - 1^{2} \\ 2 \end{bmatrix} $ or $ A = \begin{bmatrix} 3^{2} - 1^{2} \\ 2 \end{bmatrix} $ or $ A = \begin{bmatrix} 3^{2} - 1^{2} \\ 2 \end{bmatrix} $ A = 8 g) If the coin is tossed 5 times, find the probability of getting head. $ Ans n = 5, p = \frac{1}{2}, q = \frac{1}{2}, r = 1 $ $ p(r) = {}^{x}C, p^{r}q^{n-r} $ $ \therefore p(1) = {}^{5}C_{1}\left(\frac{1}{2}\right)^{1}\left(\frac{1}{2}\right)^{5-1} $ $ \therefore p(1) = \frac{5}{32} or 0.156 $ Attempt any THREE of the following:			Answer	Scheme
Ans $= xe^{x} - \int (e^{x} \cdot 1) dx$ $= xe^{x} - e^{x} + c$ $= (e)$ Evaluate $\int \tan^{2} x dx$ $= \int (\sec^{2} x - 1) dx$ $= \tan x - x + c$ $= (f)$ Find the area enclosed by the curve $y = 2x$, x-axis and the co-ordinates $x = 1$, $x = 3$ Ans $Area A = \int_{a}^{b} y dx$ $\therefore A = \int_{a}^{3} 2x dx$ $A = 2\left[\frac{x^{2}}{2}\right]_{a}^{3}$ or $A = \left[x^{2}\right]_{a}^{3}$ $A = \left[x^{2}\right]_{a}^{3}$ or $A = \left[x^{2}\right]_{a}^{3}$	1.	d)	$\int xe^{x}dx = x \int e^{x}dx - \int \left(\int e^{x}dx \cdot \frac{d}{dx} \right) dx$	1/2
$= xe^{x} - \int e^{x} dx$ $= xe^{x} - e^{x} + c$ Evaluate $\int \tan^{2} x dx$ $= \int (\sec^{2} x - 1) dx$ $= \tan x - x + c$ f) Find the area enclosed by the curve $y = 2x$, x -axis and the co-ordinates $x = 1$, $x = 3$ Ans $Area A = \int_{a}^{b} y dx$ $\therefore A = \int_{1}^{3} 2x dx$ $A = 2\left[\frac{x^{2}}{2}\right]^{3} \qquad \text{or} \qquad A = \left[x^{2}\right]^{3}$ $A = \left[\frac{3^{2}}{2} - \frac{1^{2}}{2}\right] \qquad \text{or} \qquad A = \left[3^{2} - 1^{2}\right]$ $A = 8$ g) If the coin is tossed 5 times, find the probability of getting head. Ans $n = 5, p = \frac{1}{2}, q = \frac{1}{2}, r = 1$ $p(r) = {}^{n}C_{r}p^{r}q^{n-r}$ $\therefore p(1) = {}^{5}C_{1}\left(\frac{1}{2}\right)^{1}\left(\frac{1}{2}\right)^{3-1}$ $\therefore p(1) = \frac{5}{32} \text{or} 0.156$ Attempt any THREE of the following:		Ans		1/2
e) Evaluate $\int \tan^2 x dx$ $= \int (\sec^2 x - 1) dx$ $= \tan x - x + c$ f) Find the area enclosed by the curve $y = 2x$, x -axis and the co-ordinates $x = 1$, $x = 3$ Ans $A = \int_a^b y dx$ $\therefore A = \int_1^b 2x dx$ $A = 2\left[\frac{x^2}{2}\right]_1^3 \qquad \text{or} \qquad A = \left[x^2\right]_1^3$ $A = \left[\frac{3^2}{2} - \frac{1^2}{2}\right] \qquad \text{or} \qquad A = \left[3^2 - 1^2\right]$ $A = 8$ g) If the coin is tossed 5 times, find the probability of getting head. Ans $n = 5$, $p = \frac{1}{2}$, $q = \frac{1}{2}$, $r = 1$ $p(r) = {}^n C_r p^r q^{n-r}$ $\therefore p(1) = {}^5 C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{5-1}$ $\therefore p(1) = \frac{5}{32} \qquad \text{or} \qquad 0.156$ Attempt any THREE of the following:				/2
e) Evaluate $\int \tan^2 x dx$ $= \int (\sec^2 x - 1) dx$ $= \tan x - x + c$ Find the area enclosed by the curve $y = 2x$, x-axis and the co-ordinates $x = 1$, $x = 3$ Ans $Area A = \int_a^b y dx$ $\therefore A = \int_1^2 2^2 x^2 dx$ $A = 2\left[\frac{x^2}{2}\right]_1^3 \qquad \text{or} \qquad A = \left[x^2\right]_1^3$ $A = \left[\frac{3^2}{2} - \frac{1^2}{2}\right] \qquad \text{or} \qquad A = \left[3^2 - 1^2\right]$ $A = 8$ g) If the coin is tossed 5 times, find the probability of getting head. Ans $n = 5 , p = \frac{1}{2} , q = \frac{1}{2}, r = 1$ $p(r) = {}^{p}C, p^{r}q^{n-r}$ $\therefore p(1) = {}^{5}C_1\left(\frac{1}{2}\right)^{1}\left(\frac{1}{2}\right)^{5-1}$ $\therefore p(1) = \frac{5}{32} \text{or} 0.156$ Attempt any THREE of the following:			·	1
Ans $\int \tan^2 x dx$ $= \int (\sec^2 x - 1) dx$ $= \tan x - x + c$ f) Find the area enclosed by the curve $y = 2x$, x -axis and the co-ordinates $x = 1$, $x = 3$ Ans $A = \int_1^b y dx$ $\therefore A = \int_1^3 2x dx$ $A = 2\left[\frac{x^2}{2}\right]_1^3 \qquad \text{or} \qquad A = \left[x^2\right]_1^3$ $A = \left[\frac{3^2}{2} - \frac{1^2}{2}\right] \qquad \text{or} \qquad A = \left[3^2 - 1^2\right]$ $A = 8$ g) If the coin is tossed 5 times, find the probability of getting head. Ans $n = 5, p = \frac{1}{2}, q = \frac{1}{2}, r = 1$ $p(r) = {}^n C_r p^r q^{n-r}$ $\therefore p(1) = {}^5 C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{5-1}$ $\therefore p(1) = \frac{5}{32} \qquad \text{or} \qquad 0.156$ Attempt any THREE of the following:			- xc	
f) Find the area enclosed by the curve $y = 2x$, x -axis and the co-ordinates $x = 1$, $x = 3$ Ans Area $A = \int_{a}^{b} y dx$ $A = 2\left[\frac{x^{2}}{2}\right]_{1}^{3} \qquad \text{or} \qquad A = \left[x^{2}\right]_{1}^{3}$ $A = \left[\frac{3^{2}}{2} - \frac{1^{2}}{2}\right] \qquad \text{or} \qquad A = \left[3^{2} - 1^{2}\right]$ $A = 8$ g) If the coin is tossed 5 times, find the probability of getting head. Ans $n = 5$, $p = \frac{1}{2}$, $q = \frac{1}{2}$, $r = 1$ $p(r) = {}^{n}C_{r}p^{r}q^{n-r}$ $\therefore p(1) = {}^{3}C_{1}\left(\frac{1}{2}\right)^{1}\left(\frac{1}{2}\right)^{5-1}$ $\therefore p(1) = \frac{5}{32} \qquad \text{or} \qquad 0.156$ Attempt any THREE of the following:		e)	Evaluate $\int \tan^2 x dx$	02
f) Find the area enclosed by the curve $y = 2x$, x -axis and the co-ordinates $x = 1$, $x = 3$ Ans Area $A = \int_{a}^{b} y dx$ $\therefore A = \int_{a}^{3} 2x dx$ $A = 2\left[\frac{x^{2}}{2}\right]_{1}^{3} \qquad \text{or} \qquad A = \left[x^{2}\right]_{1}^{3}$ $A = \left[\frac{3^{2}}{2} - \frac{1^{2}}{2}\right] \qquad \text{or} \qquad A = \left[3^{2} - 1^{2}\right]$ $A = 8$ g) If the coin is tossed 5 times, find the probability of getting head. Ans $n = 5$, $p = \frac{1}{2}$, $q = \frac{1}{2}$, $r = 1$ $p(r) = {}^{n}C_{r}p^{r}q^{n-r}$ $\therefore p(1) = {}^{5}C_{1}\left(\frac{1}{2}\right)^{1}\left(\frac{1}{2}\right)^{5-1}$ $\therefore p(1) = \frac{5}{32} \qquad \text{or} \qquad 0.156$ Attempt any THREE of the following:		Ans	$\int \tan^2 x dx$	
Find the area enclosed by the curve $y = 2x$, x -axis and the co-ordinates $x = 1$, $x = 3$ Area $A = \int_{1}^{h} y dx$ $\therefore A = \int_{1}^{3} 2x dx$ $A = 2\left[\frac{x^{2}}{2}\right]_{1}^{3} \qquad \text{or} \qquad A = \left[x^{2}\right]_{1}^{3}$ $A = \left[\frac{3^{2}}{2} - \frac{1^{2}}{2}\right] \qquad \text{or} \qquad A = \left[3^{2} - 1^{2}\right]$ $A = 8$ g) If the coin is tossed 5 times, find the probability of getting head. Ans $n = 5$, $p = \frac{1}{2}$, $q = \frac{1}{2}$, $r = 1$ $p(r) = {}^{n}C_{r}p^{r}q^{n-r}$ $\therefore p(1) = {}^{5}C_{1}\left(\frac{1}{2}\right)^{1}\left(\frac{1}{2}\right)^{5-1}$ $\therefore p(1) = \frac{5}{32} \qquad \text{or} \qquad 0.156$ Attempt any THREE of the following:			$= \int (\sec^2 x - 1) dx$	1
Ans $Area A = \int_{a}^{b} y dx$ $\therefore A = \int_{1}^{3} 2x dx$ $A = 2\left[\frac{x^{2}}{2}\right]_{1}^{3} \qquad \text{or} \qquad A = \left[x^{2}\right]_{1}^{3}$ $A = \left[\frac{3^{2}}{2} - \frac{1^{2}}{2}\right] \qquad \text{or} \qquad A = \left[3^{2} - 1^{2}\right]$ $A = 8$ $g) \qquad \text{If the coin is tossed 5 times, find the probability of getting head.}$ $n = 5 , p = \frac{1}{2} , q = \frac{1}{2}, r = 1$ $p(r) = {}^{n}C_{r}p^{r}q^{n-r}$ $\therefore p(1) = {}^{5}C_{1}\left(\frac{1}{2}\right)^{1}\left(\frac{1}{2}\right)^{5-1}$ $\therefore p(1) = \frac{5}{32} \qquad \text{or} \qquad 0.156$ Attempt any THREE of the following:			$=\tan x - x + c$	1
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$A = 2\left[\frac{x^2}{2}\right]^3 \qquad \text{or} \qquad A = \left[x^2\right]^3$ $A = \left[\frac{3^2}{2} - \frac{1^2}{2}\right] \qquad \text{or} \qquad A = \left[3^2 - 1^2\right]$ $A = 8$ $g) \qquad \text{If the coin is tossed 5 times, find the probability of getting head.}$ $Ans \qquad n = 5 , p = \frac{1}{2} , q = \frac{1}{2}, r = 1$ $p(r) = {}^nC_r p^r q^{n-r}$ $\therefore p(1) = {}^5C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{5-1}$ $\therefore p(1) = \frac{5}{32} \qquad \text{or} \qquad 0.156$ $Attempt any THREE of the following:$,		02
$A = 2\left[\frac{x^2}{2}\right]^3 \qquad \text{or} \qquad A = \left[x^2\right]^3$ $A = \left[\frac{3^2}{2} - \frac{1^2}{2}\right] \qquad \text{or} \qquad A = \left[3^2 - 1^2\right]$ $A = 8$ $g) \qquad \text{If the coin is tossed 5 times, find the probability of getting head.}$ $Ans \qquad n = 5 , p = \frac{1}{2} , q = \frac{1}{2}, r = 1$ $p(r) = {}^nC_r p^r q^{n-r}$ $\therefore p(1) = {}^5C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{5-1}$ $\therefore p(1) = \frac{5}{32} \qquad \text{or} \qquad 0.156$ $Attempt any THREE of the following:$		7 Mis	Area $A = \int_{a} y dx$	
$A = 2\left[\frac{x^2}{2}\right]^3 \qquad \text{or} \qquad A = \left[x^2\right]^3$ $A = \left[\frac{3^2}{2} - \frac{1^2}{2}\right] \qquad \text{or} \qquad A = \left[3^2 - 1^2\right]$ $A = 8$ $g) \qquad \text{If the coin is tossed 5 times, find the probability of getting head.}$ $Ans \qquad n = 5 , p = \frac{1}{2} , q = \frac{1}{2}, r = 1$ $p(r) = {}^nC_r p^r q^{n-r}$ $\therefore p(1) = {}^5C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{5-1}$ $\therefore p(1) = \frac{5}{32} \qquad \text{or} \qquad 0.156$ $Attempt any THREE of the following:$			$\therefore A = \int_{1}^{3} 2x dx$	1/2
g) If the coin is tossed 5 times, find the probability of getting head. Ans $n=5$, $p=\frac{1}{2}$, $q=\frac{1}{2}, r=1$ $p(r) = {}^{n}C_{r}p^{r}q^{n-r}$ $\therefore p(1) = {}^{5}C_{1}\left(\frac{1}{2}\right)^{1}\left(\frac{1}{2}\right)^{5-1}$ $\therefore p(1) = \frac{5}{32} or 0.156$ Attempt any THREE of the following:			$A = 2\left[\frac{x^2}{2}\right]_1^3 \qquad \text{or} \qquad A = \left[x^2\right]_1^3$	1/2
g) If the coin is tossed 5 times, find the probability of getting head. Ans $n=5$, $p=\frac{1}{2}$, $q=\frac{1}{2}, r=1$ $p(r) = {}^{n}C_{r}p^{r}q^{n-r}$ $\therefore p(1) = {}^{5}C_{1}\left(\frac{1}{2}\right)^{1}\left(\frac{1}{2}\right)^{5-1}$ $\therefore p(1) = \frac{5}{32} or 0.156$ Attempt any THREE of the following:			$A = \left[\frac{3^2}{2} - \frac{1^2}{2} \right]$ or $A = \left[3^2 - 1^2 \right]$	1/2
Ans $n = 5$, $p = \frac{1}{2}$, $q = \frac{1}{2}$, $r = 1$ $p(r) = {}^{n}C_{r}p^{r}q^{n-r}$ $\therefore p(1) = {}^{5}C_{1}\left(\frac{1}{2}\right)^{1}\left(\frac{1}{2}\right)^{5-1}$ $\therefore p(1) = \frac{5}{32}$ or 0.156 Attempt any THREE of the following:				1/2
$p(r) = {}^{n}C_{r}p^{r}q^{n-r}$ $\therefore p(1) = {}^{5}C_{1}\left(\frac{1}{2}\right)^{1}\left(\frac{1}{2}\right)^{5-1}$ $\therefore p(1) = \frac{5}{32} or 0.156$ Attempt any THREE of the following:		g)	If the coin is tossed 5 times, find the probability of getting head.	02
$p(r) = {}^{n}C_{r}p^{r}q^{n-r}$ $\therefore p(1) = {}^{5}C_{1}\left(\frac{1}{2}\right)^{1}\left(\frac{1}{2}\right)^{5-1}$ $\therefore p(1) = \frac{5}{32} or 0.156$ Attempt any THREE of the following:		Ans	$n=5$, $p=\frac{1}{2}$, $q=\frac{1}{2}$, $q=1$	
$\therefore p(1) = \frac{5}{32} \qquad or \qquad 0.156$				
$\therefore p(1) = \frac{5}{32} \qquad or \qquad 0.156$			$\therefore p(1) = {}^{5}C_{1}\left(\frac{1}{2}\right)^{1}\left(\frac{1}{2}\right)^{5-1}$	1
2. Attempt any THREE of the following:			$\therefore p(1) = \frac{5}{32} \qquad or \qquad 0.156$	1
			34	
Find $\frac{dy}{dx}$ if r log $y + y \log y = 0$	2.		Attempt any THREE of the following:	12
$\int \frac{dy}{dx} \int \frac{dx}{dx} dx = 0$		a)	Find $\frac{dy}{dx}$ if $x \log y + y \log x = 0$	04



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Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	a)	$x \log y + y \log x = 0$	
	Ans	$x \frac{1}{y} \frac{dy}{dx} + \log y \cdot 1 + y \frac{1}{x} + \log x \frac{dy}{dx} = 0$	2
		$\therefore \frac{dy}{dx} \left(\frac{x}{y} + \log x \right) = -\log y - \frac{y}{x}$	1
		$\therefore \frac{dy}{dx} = \frac{-\log y - \frac{y}{x}}{\frac{x}{y} + \log x}$ $\therefore \frac{dy}{dx} = \frac{y(-x\log y - y)}{x(x + y\log x)}$	1
		$\therefore \frac{dy}{dx} = \frac{y(-x\log y - y)}{x(x + y\log x)}$	
	b)	If $x = a \sec t$, $y = b \tan t$, find $\frac{dy}{dx}$ at $t = \frac{\pi}{2}$	04
	Ans	$x = a \sec t$	
		$\therefore \frac{dx}{dt} = a \sec t \tan t$	1
		$y = b \tan t$	
		$\therefore \frac{dy}{dt} = b \sec^2 t$	1
		$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{b\sec^2 t}{a\sec t \tan t}$	1/2
		$\frac{dy}{dx} = \frac{b \sec t}{a \tan t} = \frac{b \frac{1}{\cos t}}{a \frac{\sin t}{\cos t}} = \frac{b}{a} \cos ect$	1/2
		at $t = \frac{\pi}{2}$	
		$\frac{dy}{dx} = \frac{b}{a}\cos ec\left(\frac{\pi}{2}\right) = \frac{b}{a}(1)$	
		$\frac{dy}{dx} = \frac{b}{a}$	1
	c)	The rate of working of an engine is given by the expression $10V + \frac{4000}{V}$, where 'V' is the speed of the engine. Find the speed at which the rate of working is the least.	04
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Q. No.	Sub	Answer	Marking Scheme
2.	Q. N. Ans	4000	SCHEILE
2.	Alls	Let $S = 10V + \frac{4000}{V}$	
		$\therefore \frac{dS}{dV} = 10 - \frac{4000}{V^2}$	1
		$\therefore \frac{d^2S}{dV^2} = \frac{8000}{V^3}$	
			1
		consider $\frac{dS}{dV} = 0$	
		$10 - \frac{4000}{V^2} = 0$	
		$10 = \frac{4000}{V^2}$	
		V^2 $V^2 = 400$	
		V = -20 or V = 20	1
		for $V = 20$	
		$\frac{d^2S}{dV^2} = \frac{8000}{\left(20\right)^3} > 0$	1/2
		$\therefore S \text{ is least (minimum) at } V = 20$	1/2
	d)	A telegraph wire hangs in the form of a curve $y = a \log \left(\sec \left(\frac{x}{a} \right) \right)$ where 'a' is	04
		constant. Show that radius of curvature at any point is $a \sec\left(\frac{x}{a}\right)$	
	Ans	$y = a \log \left(\sec \left(\frac{x}{a} \right) \right)$	
		$y = a \log \left(\sec \left(\frac{x}{a} \right) \right)$ $\therefore \frac{dy}{dx} = a \frac{1}{\sec \left(\frac{x}{a} \right)} \sec \left(\frac{x}{a} \right) \tan \left(\frac{x}{a} \right) \left(\frac{1}{a} \right)$	1
		$\therefore \frac{dy}{dx} = \tan\left(\frac{x}{a}\right)$	
		$\therefore \frac{d^2 y}{dx^2} = \sec^2\left(\frac{x}{a}\right)\left(\frac{1}{a}\right)$	1
		∴ Radius of curvature is $\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$	



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Subject Name: Applied Mathematics	Model Answer	Subject Code: 22206

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Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	d)	$\therefore \rho = \frac{\left[1 + \tan^2\left(\frac{x}{a}\right)\right]^{\frac{3}{2}}}{\sec^2\left(\frac{x}{a}\right)\left(\frac{1}{a}\right)}$ $\therefore \rho = \frac{a\left[\sec^2\left(\frac{x}{a}\right)\right]^{\frac{3}{2}}}{\sec^2\left(\frac{x}{a}\right)}$ $\therefore \rho = \frac{a\sec^3\left(\frac{x}{a}\right)}{\sec^2\left(\frac{x}{a}\right)}$ $\therefore \rho = \frac{a\sec^3\left(\frac{x}{a}\right)}{\sec^2\left(\frac{x}{a}\right)}$	1/2
		$\therefore \rho = a \sec\left(\frac{x}{a}\right)$	1
3.	2)	Attempt any THREE of the following:	12
	a)	Find the equation of tangent and normal to the curve $4x^2 + 9y^2 = 40$ at $(1,2)$	04
	Ans	$4x^{2} + 9y^{2} = 40$ $8x + 18y \frac{dy}{dx} = 0$ $\therefore \frac{dy}{dx} = \frac{-4x}{9y}$ at $(1,2)$	1
		slope of tangent $m = \frac{dy}{dx} = \frac{-4(1)}{9(2)} = \frac{-2}{9}$ Equation of tangent $y - y_1 = m(x - x_1)$	1/2
		$y-2 = \frac{-2}{9}(x-1)$ $9y-18 = -2x+2$	1/2
		2x + 9y - 20 = 0	1/2
		slope of tangent $=\frac{-1}{m} = \frac{9}{2}$ Equation of normal is	1/2
		$y-2=\frac{9}{2}(x-1)$	1/2



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Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	a)	2y-4=9x-9 9x-2y-5=0	1/2
		If $\log(\sqrt{x^2 + y^2}) = \tan^{-1}(\frac{y}{x})$, find $\frac{dy}{dx}$	04
	Ans	$\log\left(\sqrt{x^2+y^2}\right) = \tan^{-1}\left(\frac{y}{x}\right)$	
		$\therefore \frac{1}{\sqrt{x^2 + y^2}} \times \frac{1}{2\sqrt{x^2 + y^2}} \left(2x + 2y\frac{dy}{dx}\right) = \frac{1}{1 + \left(\frac{y}{x}\right)^2} \left(\frac{x\frac{dy}{dx} - y.1}{x^2}\right)$	2
		$\frac{1}{\left(x^2+y^2\right)}\left(x+y\frac{dy}{dx}\right) = \frac{x^2}{x^2+y^2}\left(\frac{x\frac{dy}{dx}-y.1}{x^2}\right)$	1
		$\left(x + y\frac{dy}{dx}\right) = x\frac{dy}{dx} - y$	
		$y\frac{dy}{dx} - x\frac{dy}{dx} = -y - x$	
		$\frac{dy}{dx}(y-x) = -y-x$ $\frac{dy}{dx} = \frac{-y-x}{y-x}$	1
	c)	If $y = \log(x^2 e^x)$, find $\frac{dy}{dx}$	04
	Ans	$y = \log(x^2 e^x)$ $\frac{dy}{dx} = \frac{1}{x^2 e^x} (x^2 e^x + e^x 2x)$	3
		$\frac{dy}{dx} = \frac{xe^{x}(x+2)}{x^{2}e^{x}}$	
		$\frac{dx}{dy} = \frac{x^2 e^x}{x}$	1
	d)	Evaluate $\int \frac{e^{m\sin^{-1}x}}{\sqrt{1-x^2}} dx$	04



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Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	d)	$\int e^{m\sin^{-1}x}$	
	Ans	$\int \frac{e^{m\sin^{-1}x}}{\sqrt{1-x^2}} dx$	
		Put $\sin^{-1} x = t$	
		$\frac{1}{dx-dt}$	1
		$\therefore \frac{1}{\sqrt{1-x^2}} dx = dt$	1
		$=\int e^{mt} dt$	1
		$=\frac{\mathrm{e}^{mt}}{m}+c$	
			1
		$= \frac{e^{m\sin^{-1}x}}{+c}$	1
		m	1
4.		Attempt any THREE of the following:	12
	a)	Evaluate $\int \frac{1}{\sqrt{x^2 + 4x + 13}} dx$	
	,		04
	Ans	$\int \frac{dx}{\sqrt{x^2 + 4x + 13}}$	
			1
		Third term = $\frac{\left(4\right)^2}{4} = 4$	
		$= \int \frac{dx}{\sqrt{x^2 + 4x + 4 + 13 - 4}}$	
		$= \int \frac{dx}{\sqrt{(x+2)^2 + 9}}$ $= \int \frac{dx}{\sqrt{(x+2)^2 + 3^2}}$	1
		$\sqrt{(x+2)}$	
		$=\int \frac{dx}{\sqrt{(x+2)^2+3^2}}$	
		$= \log((x+2) + \sqrt{(x+2)^2 + 3^2}) + c$	2
		$\int_{-100}^{100} \left((x+2) + \sqrt{(x+2)} + 3 \right) + c$	2
	b)	Evaluate $\int \frac{1}{5+4\cos x} dx$	04
	Ans	$\int \frac{1}{5 + 4\cos x} dx$	
		Put $\tan \frac{x}{2} = t$, $\cos x = \frac{1 - t^2}{1 + t^2}$	
		$dx = \frac{2dt}{1+t^2}$	
		$1+t^2$	
	1		1



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Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	b)	$\therefore \int \frac{dx}{5 + 4\cos x} = \int \frac{1}{5 + 4\left(\frac{1 - t^2}{1 + t^2}\right)} \cdot \frac{2dt}{1 + t^2}$	1
		$= 2\int \frac{1}{t^2 + 9} dt$ $= 2\int \frac{1}{t^2 + 3^2} dt$	1
		$= 2 \times \frac{1}{3} \tan^{-1} \left(\frac{t}{3} \right) + c$	1
		$= \frac{2}{3} \tan^{-1} \left(\frac{\tan \frac{x}{2}}{3} \right) + c$	1
	c)	Evaluate $\int x \cdot \log(x+1) dx$	04
	Ans	$\int x.\log(x+1)dx$	1
		$= \log(x+1) \int x dx - \int \left(\int x dx \cdot \frac{d}{dx} \log(x+1) \right) dx$	1
		$= \log(x+1)\frac{x^2}{2} - \int \left(\frac{x^2}{2} \frac{1}{x+1}\right) dx$	
		$= \log\left(x+1\right) \frac{x^2}{2} - \frac{1}{2} \int \left(\frac{x^2}{x+1}\right) dx$ $x-1$	1
		$(x+1)x^2$	
		$-\frac{x^2+x}{-x}$	
		$\frac{-x-1}{1}$	
		$\frac{x^2}{x+1} = (x-1) + \frac{1}{x+1}$	1
		$\therefore I = \log(x+1)\frac{x^2}{2} - \frac{1}{2}\int \left((x-1) + \frac{1}{x+1}\right) dx$	
		$\therefore I = \frac{1}{2} \left(\log\left(x+1\right) x^2 - \left(\frac{x^2}{2} - x + \log\left(x+1\right) \right) \right) + c$	1



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	Sub	<u> </u>	Marking
Q. No.	Q. N.	Answer	Scheme
4.	d)	Evaluate $\int \frac{\sec^2 x}{(1+\tan x)(2+\tan x)} dx$	04
	Ans	$\int \frac{\sec^2 x}{(1+\tan x)(2+\tan x)} dx$ $\begin{vmatrix} Put & \tan x = t \\ \therefore & \sec^2 x dx = dt \end{vmatrix}$	
	11115		
		$\therefore \int \frac{1}{(1+t)(2+t)} dt$	1
		$\frac{1}{(1+t)(2+t)} = \frac{A}{1+t} + \frac{B}{2+t}$	
		1 = A(2+t) + B(1+t)	1/-
		$\therefore \text{Put } t = -1 \text{ , } A = 1$	1/2
		Put $t = -2$, $B = -1$	1/2
		$\therefore \frac{1}{(1+t)(2+t)} = \frac{1}{1+t} - \frac{1}{2+t}$	
		$\therefore \int \frac{1}{(1+t)(2+t)} dt = \int \left(\frac{1}{1+t} - \frac{1}{2+t}\right) dt$	1/2
		$= \log(1+t) - \log(2+t) + c$	1
		$= \log(1 + \tan x) - \log(2 + \tan x) + c$	1/2
		<u>OR</u>	
		$\int \frac{\sec^2 x}{(1+\tan x)(2+\tan x)} dx$ $Put \tan x = t$ $\therefore \sec^2 x dx = dt$	
		$\int \frac{1}{(1+t)(2+t)} dt$	1
		$=\int \frac{1}{t^2+3t+2} dt$	
		Third Term = $\frac{3^2}{4} = \frac{9}{4}$	1
		$= \int \frac{1}{t^2 + 3t + \frac{9}{4} - \frac{9}{4} + 2} dt$	
		$=\int \frac{1}{\left(t+\frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2} dt$	1/2
		$= \frac{1}{2\frac{1}{2}} \log \left \frac{t + \frac{3}{2} - \frac{1}{2}}{t + \frac{3}{2} + \frac{1}{2}} \right + c$	1



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Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	d)	$=\log\left \frac{t+1}{t+2}\right +c$	
		$= \log \left \frac{\tan x + 1}{\tan x + 2} \right + c$	1/2
		3/5	04
		Evaluate : $\int_0^4 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5} + \sqrt[3]{9-x}} dx$	
	Ans	$I = \int_0^4 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5} + \sqrt[3]{9-x}} dx $	
		$I = \int_0^4 \frac{\sqrt[3]{4 - x + 5}}{\sqrt[3]{4 - x + 5} + \sqrt[3]{9 - (4 - x)}} dx$	1
		$I = \int_0^4 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x} + \sqrt[3]{x+5}} dx $	
		Add (1) and (2)	
		$\therefore 2I = \int_0^4 \frac{\sqrt[3]{9-x} + \sqrt[3]{x+5}}{\sqrt[3]{9-x} + \sqrt[3]{x+5}} dx$	1/2
		$\therefore 2I = \int_0^4 1 \cdot dx$	1/2
		$\therefore 2I = \int_0^1 dx$ $\therefore 2I = \left[x\right]_0^4$	1
		$\therefore 2I - [x]_0$ $\therefore 2I = 4 - 0$	1/2
		$\therefore I = 2$	1/2
		OR	
		Replace $x \to 4-x$	
		$I = \int_0^4 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5} + \sqrt[3]{9-x}} dx$ Replace $x \to 4-x$ $\therefore x+5 \to 9-x$ $\& 9-x \to x+5$	
		$\therefore I = \int_0^4 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x} + \sqrt[3]{x+5}} dx$	1
		$\therefore 2I = \int_0^4 \frac{\sqrt[3]{9-x} + \sqrt[3]{x+5}}{\sqrt[3]{9-x} + \sqrt[3]{x+5}} dx$	1/2
		$= \int_0^4 1 \cdot dx$	1/2
		$\therefore 2I = \left[x\right]_0^4$	1
		$\therefore 2I = [x]_0$ $\therefore 2I = 4 - 0$	1/2
		$\therefore I = 2$	1/2
_			4.4
5.		Attempt any TWO of the following:	12



WINTER – 2018 EXAMINATION

Subject Name: Applied Mathematics Model Answer

Subject Code: 22206

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Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	a)	Find the area bounded by the parabola $y^2 = 9x$ and $x^2 = 9y$.	06
	Ans	$y^2 = 9x \qquad(1)$	
		$x^2 = 9y$	
		$\therefore y = \frac{x^2}{9}$	
		$\therefore \operatorname{eq}^{\mathrm{n}}.(1) \Rightarrow \left(\frac{x^2}{9}\right)^2 = 9x$	
		$\frac{x^4}{81} = 9x$	
		$\therefore x^4 = 729x$	
		$\therefore x^4 - 729x = 0$	
		$\therefore x(x^3-9^3)=0$	1
		$\therefore x = 0.9$	1
		Area $A = \int_{a}^{b} (y_1 - y_2) dx$	
		$\therefore A = \int_{0}^{9} \left(3\sqrt{x} - \frac{x^2}{9} \right) dx$	1
			2
		$\therefore A = \left(\frac{3x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^3}{27}\right)_0^9$	2
		$\therefore A = \left(\frac{3(9)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{(9)^3}{27}\right) - 0$	1
			1
		$\therefore A = 27$	1
	b)	Attempt the following:	06
	(i)	Form the differential equation by eliminating the arbitrary constants if	03
		$y = A\cos 3x + B\sin 3x$	
	Ans	$y = A\cos 3x + B\sin 3x$	
		$\therefore \frac{dy}{dx} = -3A\sin 3x + 3B\cos 3x$	1
		$\therefore \frac{d^2y}{dx^2} = -9A\cos 3x - 9B\sin 3x$	1
		$\therefore \frac{d^2y}{dx^2} = -9\left(A\cos 3x + B\sin 3x\right)$	1/2
		Paga No.	<u> </u>



WINTER – 2018 EXAMINATION

Subject Name: Applied Mathematics Model Answer

Subject Code: 22206

	_		200
Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	b)	$\frac{d^2y}{dx^2} = -9y$ $\frac{d^2y}{dx^2} + 9y = 0$	1/2
	b) (ii)	Solve: $e^{x+y}dx + e^{2y-x}dy = 0$	03
	Ans	$e^{x+y}dx + e^{2y-x}dy = 0$	
		$\therefore e^x e^y dx + e^{2y} e^{-x} dy = 0$	
		$\frac{e^x}{e^{-x}}dx = -\frac{e^{2y}}{e^y}dy$	1
		$e^{2x}dx = -e^{y}dy$	
		$\int e^{2x} dx = -\int e^{y} dy$	1
		$\frac{e^{2x}}{2} = -e^y + c$	1
	(c)	A body moves according to the law of motion is given by $\frac{d^2x}{dt^2}$ = 3t ² . Find its	06
		velocity at $t = 1 & v = 2$	
	Ans	Acceleration = $\frac{d^2x}{dt^2} = \frac{dv}{dt} = 3t^2$	1
		$\therefore dv = 3t^2 dt$	1
		$\therefore \int dv = \int 3t^2 dt$	1
		$\therefore \int dv = \int 3t^2 dt$ $\therefore v = \frac{3t^3}{3} + c$	1
		given $v = 2$ and $t = 1$	
		$\therefore c=1$	1
		$\therefore v = t^3 + 1$	1
6.		Attempt any TWO of the following:	12
	a)	Attempt the following:	06
	i)	On an average 2% of the population in an area suffer from T.B. What is the probability that out of 5 persons chosen at random from this area, at least two suffer from T.B?	03
	Ans	$n=5$, $p=2\% = \frac{2}{100} = 0.02$	
		Mean $m = np$	



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Subject Name: Applied Mathematics

Model Answer

Subject Code: 22206

5	ubject iv	ame: Applied Mathematics Model Answer Subject Code: 22	2200
Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	a) i)	$\therefore m = 5 \times 0.02 = 0.1$ $p(r) = \frac{e^{-m}m^r}{r!}$ $\therefore p(\text{atleast two}) = 1 - \lceil p(0) + p(1) \rceil$	1
		$=1 - \left[\frac{e^{-0.1} (0.1)^0}{0!} + \frac{e^{-0.1} (0.1)^1}{1!} \right]$ $= 0.0047$	1
			1
	ii)	10% of the components manufactured by company are defective. If twelve components selected at random, find the probability that atleast two will be defective.	03
	Ans	Given $p = 10\% = \frac{10}{100} = 0.1, n = 12$ and $q = 1 - p = 0.9$ $p(r) = {}^{n}C_{r}p^{r}q^{n-r}$ $p(\text{atleast two}) = 1 - \lceil p(0) + p(1) \rceil$	1
		$=1-\left[{}^{12}C_{0}\left(0.1\right)^{0}\left(0.9\right)^{12-0}+{}^{12}C_{1}\left(0.1\right)^{1}\left(0.9\right)^{12-1}\right]$	1
		= 0.3409	1
	b)	The number of road accidents met with by taxi drivers follow poisson distribution with mean 2 out of 5000 taxi in the city ,find the number of drivers. (i) Who does not meet an accident. (ii) Who met with an accidents more than 3 times. (Given e ⁻² = 0.1353)	06
	Ans	Let $N = 5000$, Mean $m = 2$ $p(r) = \frac{e^{-m}m^r}{r!}$ $(i) r = 0 \therefore p(0) = \frac{e^{-2}2^0}{0!}$	
		$(i)_{n=0}$ $: n(0) - e^{-2}2^{0}$	1
			1
		p(0) = 0.1353 Number of taxi drivers $= N \times p = 5000 \times 0.1353 = 676.5 \cong 677$	1
		(ii) More than three	
		$=1-\left[\frac{e^{-2}2^{0}}{0!}+\frac{e^{-2}2^{1}}{1!}+\frac{e^{-2}2^{2}}{2!}+\frac{e^{-2}2^{3}}{3!}\right]$	1
		$\begin{bmatrix} 0! & 1! & 2! & 3! \end{bmatrix}$ = 0.1429	1
		I .	1



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Q.	Sub	Angwan	Marking
No.	Q. N.	Answer	Scheme
6.	b)	Number of taxi drivers = $N \times p = 5000 \times 0.1429 = 714.5 \approx 715$	1
	c)	Weight of 4000 students are found to be normally distributed with mean 50 kgs and	06
		standard deviation 5 kgs. Find the number of students with weights	
		(i) less than 45 kgs	
		(ii) between 45 and 60 kgs	
		(Given: For a standard normal variate z area under the curve between $z = 0$ and $z = 1$	
		is 0.3413 and that between $z = 0$ and $z = 2$ is 0.4772)	
	Ans	Given $\bar{x} = 50$, $\sigma = 5$, $N = 4000$	
		(i) For $x = 45$, $z = \frac{x - x}{\sigma} = \frac{45 - 50}{5} = -1$	1
		$\sigma \qquad 5$ $\therefore p(\text{less than } 45) = A(\text{less than } -1)$	
		= 0.5 - A(1) $= 0.5 - 0.3413$	1
		= 0.3 - 0.3413 = 0.1587	1
		$\therefore \text{ No. of students} = N \cdot p$	
		$= 4000 \times 0.1587 = 634.8 \ i.e., 635$	1
		(ii) For $x = 45$, $z = \frac{x - \overline{x}}{\sigma} = \frac{45 - 50}{5} = -1$	
		For $x = 60$, $z = \frac{x - \bar{x}}{\sigma} = \frac{60 - 50}{5} = 2$	1
		$\therefore p(\text{ between 45 and }60) = A(-1) + A(2)$	
		=0.3413+0.4772	
		= 0.8185	1
		$\therefore \text{ No. of students} = N \cdot p = 4000 \times 0.8185$	
		= 3274	1
		<u>Important Note</u>	
		In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a	
		method other than the given herein. In such case, first see whether the method falls	
		within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.	
		meetimine with the serence of marking.	
		Paga No 14	