WINTER– 18 EXAMINATION

Subject Name: STRENGTH OF MATERIALS **Model Answer** Subject Code: **22306**

Important Instructions to examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills.
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q. N.	Answer	Marking Scheme
1		Attempt any FIVE of the following:	10
	a)	Moment of Inertia - It is the property of shape of area and can be defined as the summation of the product of ~~all~~ elementry areas and square of ~~its~~ their centroidal distances from the reference axis. — OR. — Product of area and its centroidal distance from reference axis is called as moment of area. The moment of moment of area about the reference axis is known as 'second moment of area' or moment of inertia.	01
		Unit of Moment of inertia → mm^4, or cm^4 or m^4	01
	b)	Parts subjected to tensile stresses → spokes of wheels, brake wires, clutch wires, chaindrive.	½ each Max. 01

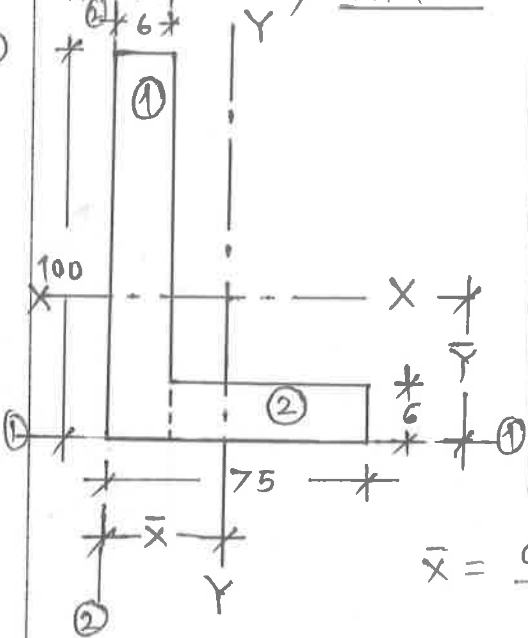
Q. No.	Sub Q. N.	Answer	Marking Scheme
		Parts subjected to compressive stresses– shock absorbers, foot rests, brake-padels.	$\frac{1}{2}$ each Max 01.
1	c)	Relationship between three moduli– $$E = \frac{9GK}{G+3K}$$	01
		Where :- E = Modulus of Elasticity G = Modulus of rigidity K = Bulk Modulus.	01
		– OR –	
		i) $E = 2G(1+\mu)$	½
		ii) $E = 3K(1-2\mu)$	½
		Where, E = Modulus of Elasticity G = Modulus of rigidity K = Bulk Modulus μ = Poisson's ratio.	01
1	d)	Shear force :- Shear force at a section of a loaded beam is defined as the net or unbalanced vertical force on either side of the section.	½
		Unit of shear force → N or kN.	½
		Bending Moment ; Bending moment at any section of the beam is the algebric summation of moments of all the vertical forces on eighter side of the section, the moments being taken about the section.	½
		Unit of bending moment → N·m or kN·m	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
1	e)	Bending equation: $$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$	01
		Where E = Modulus of Elasticity of beam material R = Radius of curvature of the beam. M = Moment of resistance I = Moment of ineria of beam c/s about N.A. σ = bending stress at the layer situated at a distance y from N.A. y = distance of layer from N.A.	01
1	f)	Eccentric load: When the line of action of the load acts parallel to but away from the axis of the member it is known as eccentric loading.	01
		Examples of eccentric loading – 1) force applied through C-clamp. 2) Compressive force acting on outer edge of circular column.	$\frac{1}{2}+\frac{1}{2}$
1	g)	Twisting moment – The moment which is applied in the plane perpendicular to the axis of the shaft is called as twisting moment. –OR– Moment produced due to tangential force about the axis of rotation of shaft is called as twisting moment.	01
		Unit of twisting moment → N-mm.	01

WINTER – 18 EXAMINATION

Subject Name: STRENGTH OF MATERIALS **Model Answer** subject Code: **22306**

Q. No.	Sub Q. N.	Answer	Marking Scheme
2		Attempt any <u>THREE</u> of the following:	12
	a)		



$a_1 = 6 \times 100 = 600\ mm^2$

$a_2 = 69 \times 6 = 414\ mm^2$

$x_1 = 6/2 = 3\ mm.$

$x_2 = 6 + 69/2 = 40.50\ mm.$

$y_1 = 100/2 = 50\ mm$

$y_2 = 6/2 = 3\ mm.$

$\bar{x} = \dfrac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = 18.31\ mm.$ from ② ② — ½

$\bar{Y} = \dfrac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = 30.81\ mm.$ from ① ① — ½

$I_{xx} = (I_{xx})_1 + (I_{xx})_2$

$= \left(\dfrac{6 \times 100^3}{12} + 600(50 - 30.81)^2\right) + \left(\dfrac{69 \times 6^3}{12} + 414(30.81 - 3)^2\right)$

$I_{xx} = 7.21 \times 10^5\ mm^4 + 3.21 \times 10^5\ mm^4$

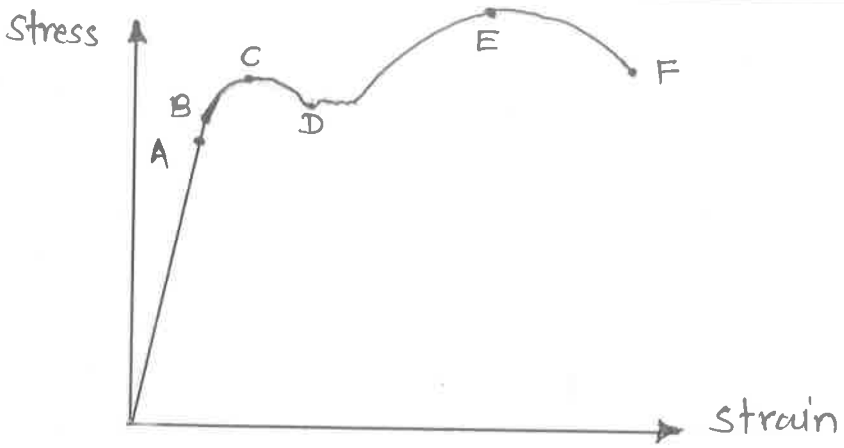
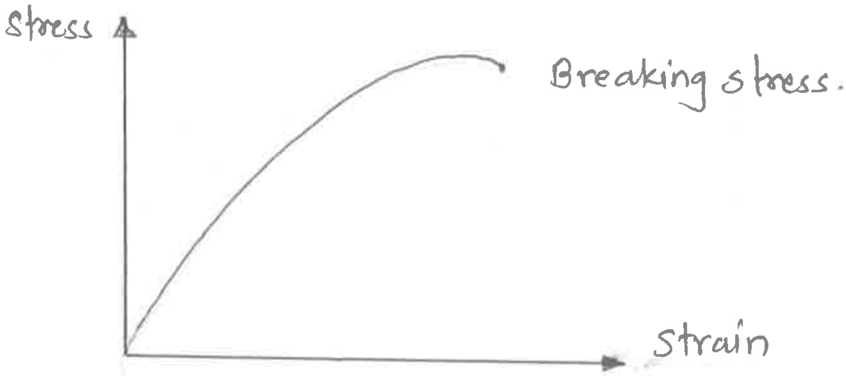
$\underline{I_{xx} = 10.42 \times 10^5\ mm^4}$ — 1½

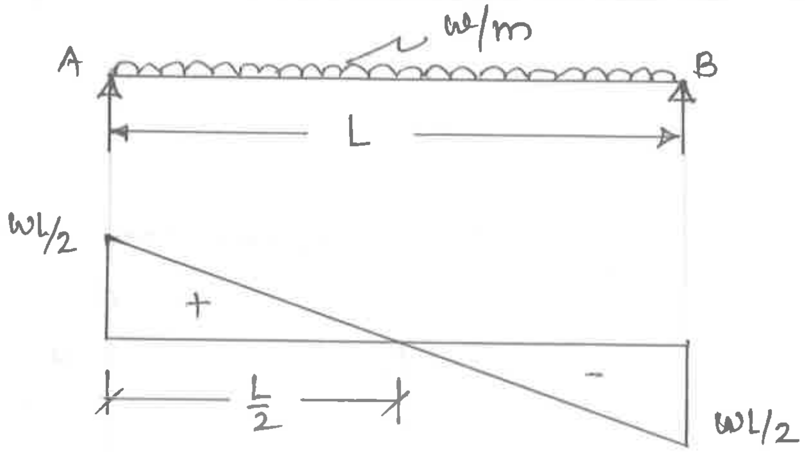
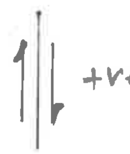
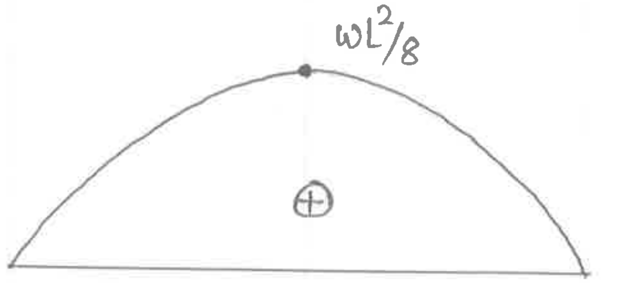
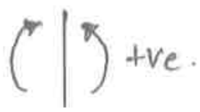
$I_{YY} = (I_{yy})_1 + (I_{yy})_2$

$= \left[\dfrac{100 \times 6^3}{12} + 600(18.31 - 3)^2\right] + \left[\dfrac{6 \times 69^3}{12} + 414(40.50 - 18.31)^2\right]$

$= 1.42 \times 10^5 + 3.68 \times 10^5$

$\underline{I_{YY} = 5.1 \times 10^5\ mm^4}$ — 1½

Q. No.	Sub Q. N.	Answer	Marking Scheme
2	b		02
		• Stress strain diagram for mild steel. A — Proportionality limit B — Elastic limit C — Upper yield point D — lower yield point E — Ultimate stress point F — Breaking stress (nominal) point.	01
		 Stress-strain variation for cast iron.	01

Q. No.	Sub Q. N.	Answer	Marking Scheme
2	c	Given – $L = 300\,mm$, $b = 40\,mm$, $d = 40\,mm$, $P = 400 \times 10^3\,N$ $\delta L = 0.075\,cm = 0.75\,mm$, $\delta b = 0.03\,mm$. Solution – $\text{stress} = \sigma = \frac{P}{A} = \frac{400 \times 10^3}{40 \times 40} = 250\,N/mm^2$	01
		$\text{strain} = e = \frac{\delta L}{L} = \frac{0.75}{300} = 2.5 \times 10^{-3}$ $\text{Lateral strain} = e_{Lat} = \frac{\delta d}{d} = \frac{0.03}{40} = 7.5 \times 10^{-4}$	01
		$\text{Young's Modulus} = E = \frac{\sigma}{e} = \frac{250}{2.5 \times 10^{-3}} = 1 \times 10^5\,N/mm^2$	01
		$\text{Poisson's ratio} = \mu = \frac{e_{Lat}}{e_{Lin}} = \frac{7.5 \times 10^{-4}}{2.5 \times 10^{-3}} = 0.30$	01
2	d	 Shear force diagram.	02
		 Bending moment diagram.	02

Q. No.	Sub Q. N.	Answer	Marking Scheme
3		Attempt any THREE of the following.	12
	a	Given - for solid rectangular section, b= 40mm, d= 60mm.	

Solution:

for solid rectangular section-

$$I_{base} = I_G + Ay^2 \qquad 1$$

$$= \frac{bd^3}{12} + b\cdot d\,(d/2)^2$$

$$= \frac{40\times60^3}{12} + 40\times60\times\left(\frac{60}{2}\right)^2 \qquad 1$$

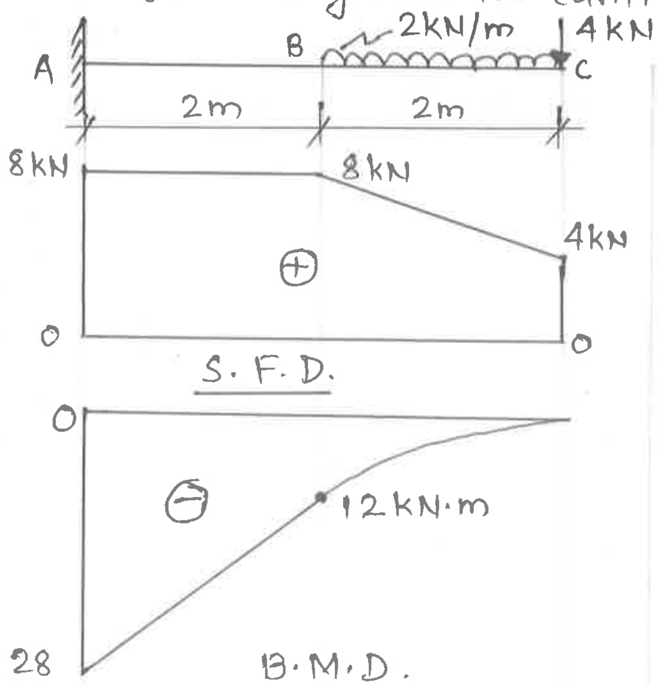
$$= 7.20\times10^5 + 21.6\times10^5$$

$$I_{base} = 28.8\times10^5\ mm^4 \qquad 2$$

- OR -

$$I_{base} = \frac{bd^3}{3} = \frac{40\times60^3}{3} = 28.8\times10^5\ mm^4 \qquad 2+2$$

Q. No.	Sub Q. N.	Answer	Marking Scheme
3	b	S.F & B.M. diagrams for cantilever beam.	



S.F. Calculations ↑|↓ +ve. — 1

$S.F_C = 4\ kN.$

$S.F_B = 4 + (2\times2) = 8\ kN.$ — SFD 1

$S.F_A = 8\ kN.$

B.M. Calculations ↻|↺ +ve. — 1

$BM_C = 0$

$BM_B = -4\times2 - (2\times2)\times1$ — BMD-1

$= -12\ kN\cdot m.$

$BM_A = -4\times4 - 2\times2\times3$

$= -28\ kN\cdot m.$

Q. No.	Sub Q. N.	Answer	Marking Scheme
3	C	Given – for solid shaft, $d = 110\,mm$, $R = 55\,mm$, $T = 12.5\,kN{\cdot}m = 12.5\times10^6\,N{\cdot}mm$, $L = 2.5\,m = 2500\,mm$, $G = 80\,GPa = 80\times10^3\,N/mm^2$.	
		Solution –	
		Polar M.I $= I_p = \frac{\pi}{32}d^4 = \frac{\pi}{32}\times110^4 = 14.37\times10^6\,mm^4$	01
		i) Using the relation, $\frac{T}{I_p} = \frac{q_{max}}{R}$	
		$\therefore q_{max} = \frac{T}{I_p}\times R = \frac{12.5\times10^6\times55}{14.37\times10^6}$	
		$\therefore q_{max} = 47.84\,N/mm^2$	1½
		ii) Using the relation, $\frac{T}{I_p} = \frac{G\theta}{L}$	
		$\therefore \theta = \frac{T}{I_p}\times\frac{L}{G} = \frac{12.5\times10^6\times2500}{14.37\times10^6\times80\times10^3}$	
		$\theta = 0.02718$ radians.	1½
3	d	Given – for offset link, $d = 30\,mm$, $\sigma_{t\,max} = 80\,N/mm^2$, eccentricity $= e = 40+\frac{d}{2} = 40+\frac{30}{2} = 55\,mm$.	01
		Solution:-	
		C/s Area $= A = \frac{\pi}{4}\times d^2 = \frac{\pi}{4}\times30^2 = 706.86\,mm^2$	
		M.I $= I = \frac{\pi}{64}\times d^4 = \frac{\pi}{64}\times30^4 = 39.76\times10^3\,mm^4$	
		Section Modulus $= Z_1 = I/y_{max} = \frac{39.76\times10^3}{15} = 2650.7\,mm^3$	01
		Using the relation	
		$\sigma_{max} = \sigma_0 + \sigma_b = \frac{P}{A} + \frac{P.e}{Z_1}$	01
		$\therefore 80 = \frac{P}{706.86} + \frac{P\times55}{2650.7}$	
		$\therefore P = 3610.1\,N$	01

Q. No.	Sub Q. N.	Answer	Marking Scheme
4		Attempt any THREE of the following	12

a) Reactions:

$\Sigma M_A = 0$ | ↻ +ve.

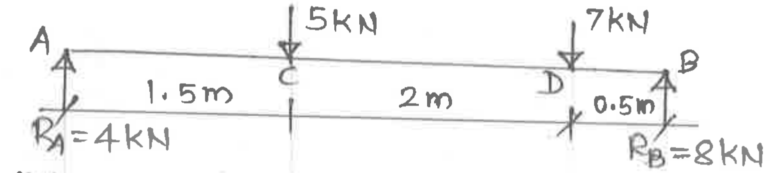
$5 \times 1.5 + 7 \times 3.5 - R_B \times 4 = 0$

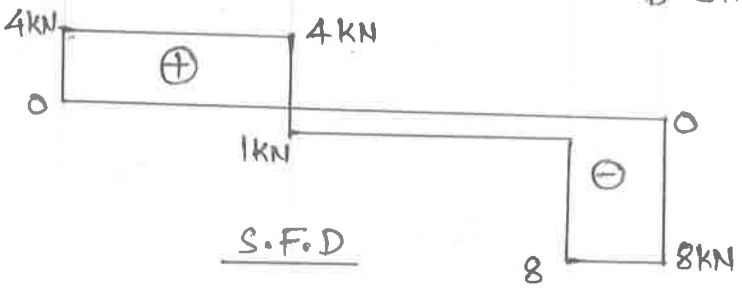
$\therefore R_B = 8\ kN.$

$\Sigma F_y = 0$ | ↑ +ve.

$R_A + R_B - 5 - 7 = 0$

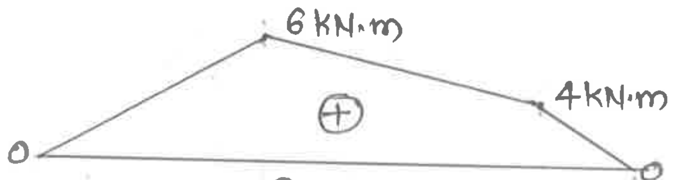
$\therefore R_A = 12 - 8 = 4\ kN.$





↑|↓ +ve

S.F.D — 01



↻|↺ +ve.

BMD — 01

S.F. Calculations ↑|↓ +ve.

$S.F_A = 4\ kN$

$S.F_C$ (left) = 4kN.

$S.F_C$ (right) 4−5 = −1kN.

SF_D (left) = −1 kN.

SF_D right = −1−7 = −8kN

SF_B = −8 kN.

B.M. calculations ↻|↺ +ve — 01 + 01

$BM_A = BM_B = 0$

$BM_C = 4 \times 1.5 = 6\ kN.m.$

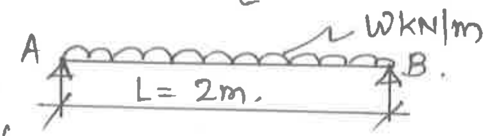
$BM_D = 4 \times 3.5 - 5 \times 2$

$= 4\ kN.m.$

Q. No.	Sub Q. N.	Answer	Marking Scheme
4	b		

Given, for rectangular s.s. beam, $b = 100\ mm$, $d = 150\ mm$.
$L = 2\ m$, $\sigma_b = 28\ N/mm^2$, $q_{max} = 2\ N/mm^2$

Solution :



$$\text{Max. B.M} = M = \frac{wL^2}{8} \times 10^6\ Nmm$$

$$\text{Max S.F} = S = R_A = \frac{wL}{2} \times 10^3\ N$$

for beam section, $I = \frac{bd^3}{12} = \frac{100 \times 150^3}{12} = 28.125 \times 10^6\ mm^4$

$$Y_{max} = d/2 = 150/2 = 75\ mm.$$

½

1) Value of 'w' for bending stress criteria.

$$M = \frac{\sigma}{y} \times I$$

$$10^6 \times \frac{wL^2}{8} = \frac{28 \times 28.125 \times 10^6}{75}$$

$$10^6 \times \frac{w \times 2^2}{8} = \frac{28 \times 28.125 \times 10^6}{75}$$

$\therefore w = 21\ kN/m$ ——— (A) 1½

2) Value of 'w' for shear stress criteria.

$$q_{max} = 1.5 \frac{S}{A}$$

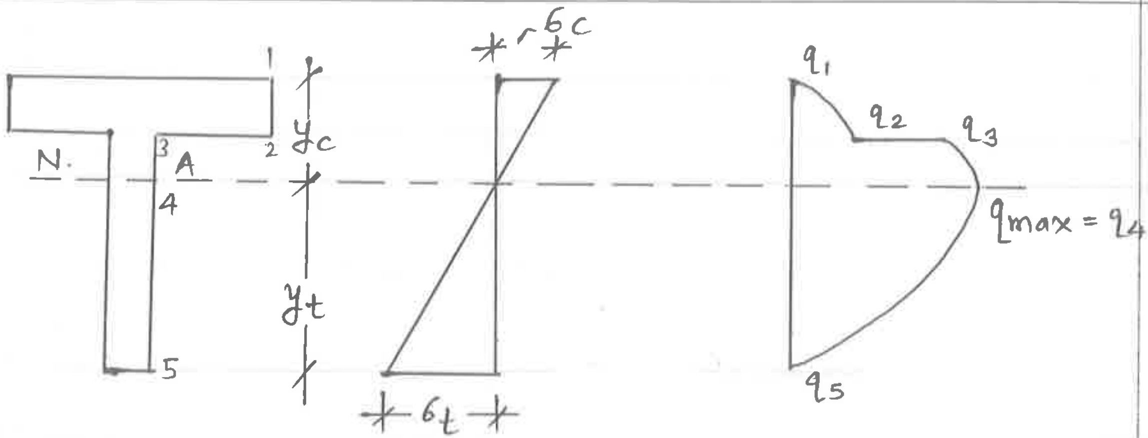
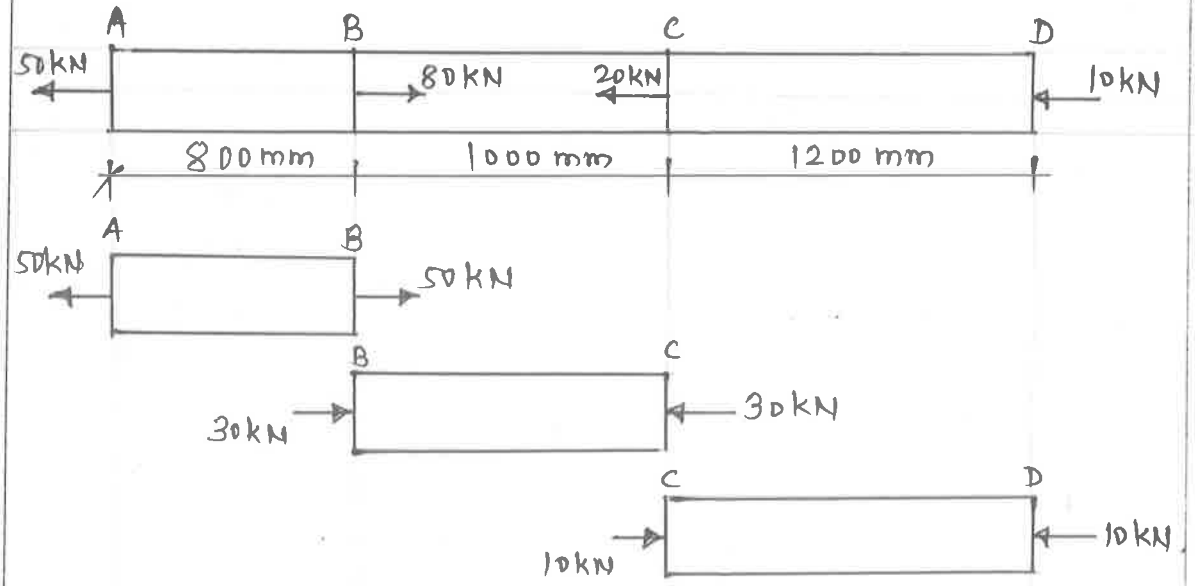
$$2 = \frac{1.5 \times w \times 2 \times 10^3}{2 \times 100 \times 150}$$

$\therefore w = 20\ kN/m.$ ——— (B) 1½

$\therefore$ Permissible UDL = minimum of (A) & (B)

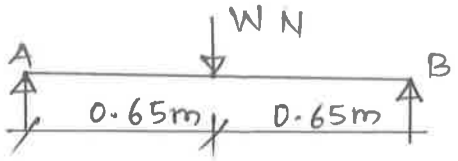
$\therefore w = 20\ kN/m.$ ½

Q. No.	Sub Q. N.	Answer	Marking Scheme
4	c	Given : for solid circular shaft $d = 40\,mm$, $N = 200\,rpm$, $q_{max} = 85\,N/mm^2$ Solution for solid shaft, $I_p = \frac{\pi}{32} d^4 = \frac{\pi}{32} \times 40^4 = 2.51 \times 10^5\,mm^4$ $R = d/2 = 40/2 = 20\,mm.$	01
		Using the relation, $\frac{T}{I_p} = \frac{q_{max}}{R}$ $\therefore T = \frac{q_{max} \times I_p}{R} = \frac{85 \times 2.51 \times 10^5}{20} = 1.07 \times 10^6\,N{\cdot}mm$ $T = 1.07 \times 10^3\,N{\cdot}m$ Assuming $T_{max} = T_{avg} = 1.07 \times 10^3\,N{\cdot}m.$	$1\frac{1}{2}$
		Power $= \frac{2\pi N T_{avg}}{60} = \frac{2\pi \times 200 \times 1.07 \times 10^3}{60}$ $= 22410.03$ Watts. $P = 22.41\,kW.$	$1\frac{1}{2}$
4	d	Given for M.S. link, $P = 80\,kN = 80 \times 10^3\,N$, $b = 3t$, $\sigma = 70\,N/mm^2$ Solution: $\sigma = \frac{P}{A}$ $\therefore A = \frac{P}{\sigma} = \frac{80 \times 10^3}{70}$	01
		$\therefore A = 1142.86\,mm^2$	01
		$\therefore b \times t = 1142.86$ $\therefore 3t \times t = 1142.86$ $\therefore t = 19.52\,mm$	01
		& $b = 3t = 3 \times 19.52 = 58.56\,mm.$	01

Q. No.	Sub Q. N.	Answer	Marking Scheme
4	e	 C/s of beam Bending stress distribution Shear stress distribution.	02+02
Q5		Attempt any <u>TWO</u> of the following:	12
	a	Given, for brass bar, $A = 1000\ mm^2$, $E = 1.05 \times 10^5\ N/mm^2$ Solution:-  Equilibrium of individual parts-	02

Q. No.	Sub Q. N.	Answer	Marking Scheme
5	a contd.	i) $\delta L_{AB} = \left(\frac{PL}{AE}\right)_{AB} = \frac{50\times10^3\times800}{1000\times1.05\times10^5} = +0.381$ mm.	01
		ii) $\delta L_{BC} = \left(\frac{PL}{AE}\right)_{BC} = -\frac{30\times10^3\times1000}{1000\times1.05\times10^5} = -0.286$ mm.	01
		iii) $\delta L_{CD} = \left(\frac{PL}{AE}\right)_{CD} = -\frac{10\times10^3\times1200}{1000\times1.05\times10^5} = -0.114$ mm.	01
		Net deformation $= \delta_{AB} + \delta_{BC} + \delta_{CD}$ $= 0.381 - 0.286 - 0.114$ $\delta L = -0.019$ mm. (–ve sign indicates decrease in length)	01
5	b	On the basis of mechanical properties given materials can be arranged in decreasing order as below.	
		Ⓐ Criteria – strength. 1) Mild steel 2) Copper 3) Wood 4) leather.	02
		Ⓑ Criteria – Hardness — 1) Mild steel. 2) Copper 3) Wood 4) leather.	02
		© Criteria – Ductility – 1) Copper 2) Mild steel 3) leather 4) Wood.	02

Q. No.	Sub Q. N.	Answer	Marking Scheme
5	c	Given – For short circular (hollow) column $D = 40\,cm = 400\,mm$ $d = 20\,cm = 200\,mm.$ Criteria – no tension at base.	
		Solution	
		C/s Area $= A = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}(400^2 - 200^2) = 94.25\times10^3\,mm^2$	01
		$M.I = I = \frac{\pi}{64}(D^4 - d^4) = \frac{\pi}{64}(400^4 - 200^4) = 11.78\times10^8\,mm^4$	
		$y_{max} = \frac{D}{2} = \frac{400}{2} = 200\,mm.$	01
		for no tension condition – $\sigma_0 = \sigma_b$	01
		$\therefore \frac{P}{A} = \frac{P \cdot e \cdot y_{max}}{I}$	01
		$\therefore e = \frac{I}{A \times y_{max}} = \frac{11.78\times10^8}{94.25\times10^3\times200}$	01
		$\therefore e = 62.49\,mm.$	01

Q. No.	Sub Q. N.	Answer	Marking Scheme
6		Attempt any TWO of the following:	12
	a	Given for a rectangular S.S. wooden beam – $b = 150 mm$, $d = 250 mm$, $L = 1.3 m$. Central point load = 'W' N $\sigma_b = 7 N/mm^2$ and $q_{max} = 1 N/mm^2$	
		Solution 	
		$M = \text{Max B.M} = \frac{WL}{4} = \frac{W \times 1.3}{4} = 0.325 W \cdot N \cdot m$ $M = 325 W \; N \cdot mm$.	01
		$S = \text{Max S.F} = \text{Reaction} = \frac{W}{2} N = 0.5 W N$.	01
		for rectangular section, $A = b \times d = 150 \times 250 = 37500 mm^2$ $I = \frac{bd^3}{12} = \frac{150 \times 250^3}{12} = 195.31 \times 10^6 mm^4$ $y_{max} = d/2 = \frac{250}{2} = 125 mm$.	
		i) Value of 'W' for bending stress criteria $\frac{M}{I} = \frac{\sigma}{y}$ ∴ $M = \frac{\sigma}{y} \times I$ ∴ $325 W = \frac{7 \times 195.31 \times 10^6}{125}$ ∴ $W = 33653.41 N = 33.65 kN$ — (A)	$1\frac{1}{2}$
		ii) Value of 'W' for shear stress criteria. $q_{max} = \frac{1.5 S}{A}$ ∴ $1 = 1.5 \times 0.5 W / 37500$ ∴ $W = 50000 N = 50 kN$ — (B)	$1\frac{1}{2}$
		∴ Safe value of W = min. of (A) & (B) = 33.65 kN.	01

Q. No.	Sub Q. N.	Answer	Marking Scheme
6	b	Given, For solid circular steel shaft -	
		$P = 90 kW = 90 \times 10^3$ Watts, $N = 160$ rpm.	
		$q_{max} = 60 N/mm^2$, $G = 8 \times 10^4 N/mm^2$	
		$\theta = 1^\circ = 1 \times \frac{\pi}{180} = 0.0175$ rad.	
		Solution:- $P = \frac{2\pi N T_{avg}}{60}$	
		$90 \times 10^3 = 2\pi \times 160 \times T_{avg}/60$	
		$\therefore T_{avg} = 5.371 \times 10^3$ N·m $= 5.371 \times 10^6$ N·mm.	01
		Student may assume $T_{max} = T_{avg} = 5.371 \times 10^6$ N·mm.	
		Using the relation, $\frac{T}{I_p} = \frac{q_{max}}{R}$	01
		$\therefore \frac{5.371 \times 10^6 \times 32}{\pi d^4} = \frac{60 \times 2}{d}$	
		$\therefore d^3 = \frac{5.371 \times 10^6 \times 32}{\pi \times 60 \times 2} = 455.90 \times 10^3$	
		$\therefore d = 76.96$ mm.	01
		Now, $I_p = \frac{\pi}{32} \times d^4 = \frac{\pi}{32} \times 76.96^4 = 3.444 \times 10^6 mm^4$	01
		Using the relation, $\frac{T}{I_p} = \frac{G\theta}{L}$	01
		$\therefore L = \frac{G\theta \cdot I_p}{T} = \frac{8 \times 10^4 \times 0.0175 \times 3.444 \times 10^6}{5.371 \times 10^6}$	
		$\therefore L = 897.71$ mm	
		$\therefore$ length of shaft $= L = 0.897$ m Say 0.9 m.	01
		(Note - Student may assume $T_{max} = 1.2$ to $1.4\ T_{avg}$. Marks shall be awarded accordingly.)	

Q. No.	Sub Q. N.	Answer	Marking Scheme
6	C		

Given- For rectangular column section –
$b = 200mm$, $d = 100\,mm$, $P = 180 \times 10^3 N$.
$e = 100\,mm$ in the plane bisecting thickness

Solution:
eccentricity about YY-axis. } 01

$\therefore I_{yy} = \frac{100 \times 200^3}{12} = 66.67 \times 10^6\ mm^4$

$Y_{max} = 200/2 = 100\ mm.$

$A = 200 \times 100 = 20000 mm^2$

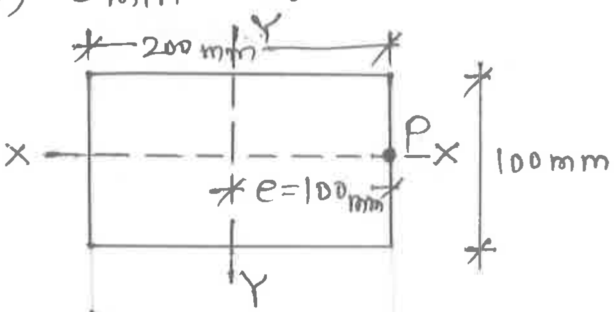
i) direct stress $= \sigma_0 = \frac{P}{A} = \frac{180 \times 10^3}{20000} = 9\ N/mm^2$ — 01

ii) bending stress $= \sigma_b = \pm \frac{P \cdot e \cdot y_{max}}{I}$ — 01

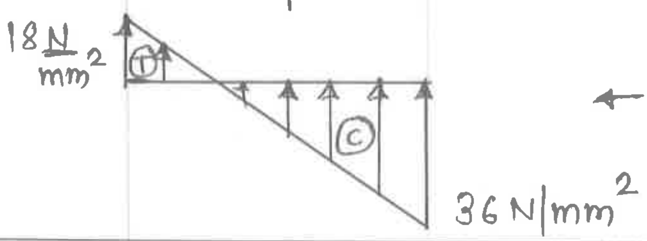
$= \pm \frac{180 \times 10^3 \times 100 \times 100}{66.67 \times 10^6}$

$\sigma_b = \pm 27\ N/mm^2$ — 01

iii) $\sigma_{max} = \sigma_0 + \sigma_b = 9 + 27 = 36\ N/mm^2$ (Comp.)
iv) $\sigma_{min} = \sigma_0 - \sigma_b = 9 - 27 = -18\ N/mm^2$
$= 18\ N/mm^2$ (Tensile). } 01



← c/s of column.



← Combined stress distribution dia — 01